

Ejercicios de Tarea 5.

1 (iii) Calcular $P_{3,0}(x)$ para $f(x) = e^{e^x}$.

$$P_{3,0}(x) = f(0) + f'(0)(x-0) + \frac{f''(0)(x-0)^2}{2!} + \frac{f'''(0)(x-0)^3}{3!}$$

$$f(x) = e^{e^x} \quad (= e^{g(x)})$$

$$f'(x) = e^{e^x} \cdot e^x$$

$$f''(x) = e^{e^x} \cdot e^x \cdot e^x + e^{e^x} \cdot e^x = e^{e^x+2x} + e^{e^x+x}$$

$$\begin{aligned} f'''(x) &= e^{e^x+2x} \cdot (e^x+2) + e^{e^x+x} \cdot (e^x+1) \\ &= e^{e^x+3x} + 2 \cdot e^{e^x+2x} + e^{e^x+2x} + e^{e^x+x} \\ &= e^{e^x+3x} + 3e^{e^x+2x} + e^{e^x+x} \end{aligned}$$

$$f(0) = e$$

$$f'(0) = e$$

$$f''(0) = e + e$$

$$= 2e$$

$$f'''(0) = e + 3e$$

$$+ e$$

$$= 5e$$

$$P_{3,0}(x) = e + e \cdot x + \frac{2e}{2} \cdot x^2 + \frac{5e}{6} \cdot x^3.$$

$$R_{3,0}(x) = \frac{f^{(4)}(\xi_x)}{4!} (x-0)^4 \quad \text{con } \xi_x \in (0, x).$$

$$f^{(4)}(x) = e^{e^x+3x} \cdot (e^x+3) + 3e^{e^x+2x} (e^x+2) + e^{e^x+x} (e^x+1).$$

$$\therefore R_{3,0}(x) = \frac{\left(e^{e^{\xi}+3\xi} (e^{\xi}+3) + 3e^{e^{\xi}+2\xi} (e^{\xi}+2) + e^{e^{\xi}+\xi} (e^{\xi}+1) \right) x^4}{4!}$$

es el residuo en la forma de Lagrange.

Luego:

$$R_{3,0}(x) = \int_0^x \frac{f^{(4)}(t)}{3!} \cdot (x-t)^3 dt.$$

$$= \int_0^x \left(e^{t+3t} (e^t+3) + 3 e^{e^t+2t} (e^t+2) + e^{e^t+t} (e^t+1) \right) \cdot (x-t)^3 dt.$$