Tarea 5.

1 (i) En $\mathbb{R}$ definimos la relación $\sim$ como

$$x \sim y \iff x^2 - x = y^2 - y.$$

Notemos que $x^2 - x = y^2 - y$

$$\iff x^2 - x + \frac{1}{4} - \frac{1}{4} = y^2 - y + \frac{1}{4} - \frac{1}{4}$$

$$\iff \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} = \left(y - \frac{1}{2}\right)^2 - \frac{1}{4}$$

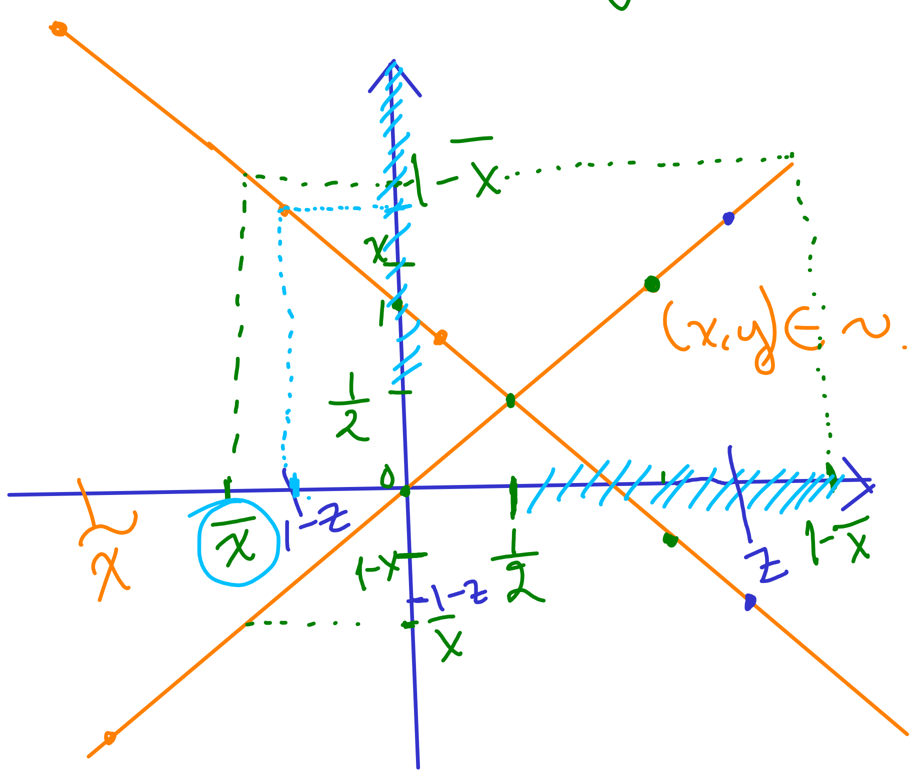
$$\iff \left(x - \frac{1}{2}\right)^2 = \left(y - \frac{1}{2}\right)^2.$$

$$\iff \left|x - \frac{1}{2}\right| = \left|y - \frac{1}{2}\right|$$

$$\iff \left(x-\tfrac{1}{2}=y-\tfrac{1}{2}\right) \text{ o } \left(x-\tfrac{1}{2}=\tfrac{1}{2}-y\right)$$

$$\text{o } \left(\tfrac{1}{2}-x=y-\tfrac{1}{2}\right) \text{ o } \left(\tfrac{1}{2}-x=\tfrac{1}{2}-y\right).$$

$$\iff x=y \quad \text{o} \quad y=1-x$$



$$[x]=\{x,\,1-x\}.$$

$$\mathbb{R}/\!\sim\; = \{[x] : x\in\mathbb{R}\}$$

$$=\{[x] : x\in(-\infty,\tfrac{1}{2}]\}$$

Nótese que $x = 1-x \iff 2x = 1 \iff x = \frac{1}{2}$

Obs. que $x < \frac{1}{2} \iff -x > -\frac{1}{2} \iff 1-x > 1-\frac{1}{2}$

$= \frac{1}{2}.$

$\boxed{x < \frac{1}{2} \iff 1-x > \frac{1}{2}}$; $\boxed{z > \frac{1}{2} \iff 1-z < \frac{1}{2}}$

$[x] = \{x, 1-x\}$

Los representantes posibles para $[x]$ son x y $1-x$.

Af: Dado $z \in \mathbb{R}$, $\exists\, x \in (-\infty, \frac{1}{2}]$ t.q. $z = x$ ó $z = 1-x$.

Caso 1: $z \in (-\infty, \frac{1}{2}]$, ent. $x = z$ cumple la condición.

Caso 2: $z \in (\frac{1}{2}, \infty)$, ent. $x = 1 - z$ es tal que como $z > \frac{1}{2}$, ent. $1 - z < \frac{1}{2}$.

$\therefore$ $x = 1 - z$ cumple la condición. $\square$